## Introduction to Bayes theorem

- For the first problem in the video - the one about the female engineers - is the quantity you are being asked to calculate a conditional or an absolute probability?
- For the female engineers problem you are given two conditional probabilities and one absolute probability. What are these the probabilities of?
- For the question about the medical test there are two random variables: there is a Bernoulli random variable that tells you whether you have the disease and a Bernoulli random variable that tells you whether or not the test result was positive. Are these random variables independent of each other? Explain your reasoning.
- Give a statement of Bayes theorem.


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A joined up approach to

- Suppose you now administered two tests to the patient to determine whether or not they have the disease. Test A comes up positive in $P\left(T_{1}=1 \mid D=1\right)$ percent of diseased patients and positive in $P\left(T_{1}=1 \mid D=0\right)$ percent of healthy patients. Test B by contrast comes up positive in $P\left(T_{2}=1 \mid D=\right.$ 1) percent of diseased patients and positive in $P\left(T_{2}=1 \mid D=0\right)$ percent of healthy patients. Draw a Venn diagram showing the various possible categories each individual could be in. If we can calculate the probability that a person has the disease given they had positive results for the two tests using $P\left(D=1 \mid T_{1}=1 \wedge T_{2}=1\right)=\frac{P\left(T_{1}=1 \mid D=1\right) P\left(T_{2}=1 \mid D=1\right) P(D=1)}{P\left(T_{1}=1 \mid D=1\right) P\left(T_{2}=1 \mid D=1\right) P(D=1)+P\left(T_{1}=0 \mid D=1\right) P\left(T_{2}=1 \mid D=1\right) P(D=1)+P\left(T_{1}=1 \mid D=1\right) P\left(T_{2}\right.}$ what further assumption have we made? How would we use Bayes theorem to calculate this conditional probability with this assumption relaxed?
- Consider a pair of discrete random variables, $X$ and $Y$, that can both take values between 0 and $n$. Use Bayes theorem to derive an expression for the conditional probability $P(X=a \mid Y=y)$ in terms of the set of absolute probabilities, $P\left(X=x_{j}\right)$, for getting each possible value for $X$ and the set of conditional probabilities $P\left(Y=y \mid X=x_{j}\right)$ for getting $Y=y$ given that $X=x_{j}$. In doing this you will need to use summation notation.

