

An introduction to Monte Carlo integration

0.1 Level 1

This exercise should be revision: Use the block below to generate 20 pairs of uniform random variables X_i and Y_i that each lie between 0 and 1. Then plot each of these pairs of random variables at (X_i, Y_i) . All the points you show should have $0 < X_i < 1$ and $0 < Y_i < 1$.

0.2 Level 2

Now use the blocks below to generate 25 points on the graph shown on the right. These points should be at $(0.1 + i * 0.2, 0.1 + j * 0.2)$ with $i \in \{0, 1, 2, 3, 4\}$ and $j \in \{0, 1, 2, 3, 4\}$. [Click here](#) if you want to watch the explanatory video.

0.3 Level 3

Use the blocks below to generate 25 points on the graph shown on the right. These points should again be at $(0.1 + i * 0.2, 0.1 + j * 0.2)$ with $i \in \{0, 1, 2, 3, 4\}$ and $j \in \{0, 1, 2, 3, 4\}$. Now though I only want you to display the points if they lie within a circle of radius one that is centered on the origin. [Click here](#) if you want to watch the explanatory video.

0.4 Level 4

Let's now reintroduce the random variables. Select a random point (X, Y) by generating two uniform random variables between 0 and 1, X and Y , and determine whether the point you selected is within a circle of radius one that is centred on the origin. Set the bernoulli random variable Z equal to one if (X, Y) is within the circle and zero otherwise. Use the blocks below to generate a sample of Z values and draw a graph showing how the sample mean, $\mu_n = \frac{1}{n} \sum_{i=1}^n Z_i$ for the random variable Z changes as the number of independent random variables in the sample (the number of Z_i values) increases. In other words, plot a graph with points at (n, μ_n) for n values between 0 and 20. [Click here](#) if you want to watch the explanatory video.

0.5 Level 5

Compute a set of 10 samples of the integral that you computed during the last exercise. Calculate each estimate of the sample mean by generating 5 Z values. Use the plotting to display the mean of your 10 samples of the integral together with an error bar indicating the 90% confidence interval. [Click here](#) if you want to watch the explanatory video.

0.6 Level 6

Compute the integral that you computed during the last exercise by sampling only once. Calculate the 80% confidence interval. [Click here](#) if you want to watch the explanatory video.