



• Complete the following sentence: If all the elements in the transition matrix of a Markov chain are positive all the states in the chain are ..."

• Explain the meaning of the terms eigenvalue and eigenvector and write an equation involving a square matrix, one of its eigenvectors and one of its eigenvalues.

• How is the inverse of a matrix defined? Can a non-square matrix have an inverse?

• Explain what we mean when we state that a matrix is diagonalisable? Can non-square matrices be diagonalized?



• Explain how you would go about calculating the fourth power of a diagonal matrix?

• The Perron-Frobeneius theorem tells us that square matrices with all positive real elements always have a unique largest eigenvalue,  $\lambda_1$ . What does this tell us about the following limit  $\lim_{n\to\infty} \left(\frac{\lambda_2}{\lambda_1}\right)^n$  if  $\lambda_2$  is the second largest eigenvalue?

• Consider a  $2 \times 2$  diagonal matrix, **A**, that has  $A(1,1) = \lambda$  and calculate the following product of three matrices **VAB**. Use  $v_{11}$ ,  $v_{12}$ ,  $v_{21}$  and  $v_{22}$  for the elements of **V** and similar names for the elements of **B**. What do you notice about the rows of the final product matrix. Explain the significance of this result in the context of Markov chains.